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A block effect of external field in the one-dimensional ferromagnetical Ising model with long-range interaction

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Abstract
We consider the one-dimensional ferromagnetical Ising model with long-range interaction under external field blocks of equal length with alternating signs and investigate the low-temperature phase diagram of this model. It turns out that when the absolute value of the external field is sufficiently small, the set of Gibbs states substantially depends on block size: at small block sizes there are at least two Gibbs states and at large block sizes there is a unique Gibbs state.

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1. Introduction

Consider the one-dimensional ferromagnetical Ising model with long-range interaction:

$$H_0(\phi) = -\sum_{x,y\in\mathbf{Z}^1;x>y} U(x-y)\phi(x)\phi(y) \tag{1}$$

where the spin variables $\phi(x)$ associated with the one-dimensional lattice sites x take values from the set $\{-1, 1\}$ and the pair potential $U(x-y) = (x-y)^{-\gamma}$, $1 < \gamma \leqslant 2$. The condition $\gamma > 1$ is necessary for the existence of the thermodynamical limit. We focus on the case $\gamma \leqslant 2$, otherwise $\sum_{x\in\mathbf{Z}^1,x>0} xU(x) < \infty$ and the model (1) has a unique Gibbs state [1–3].

Dyson considered a model (1) with a positive pair potential $U(r) = U(|x-y|)$ satisfying the conditions [4, 5]:

(1) $\sum_{r=1}^{\infty} U(r) < \infty$,
(2) $U(r) > U(r+1)$,
(3) $\sum_{r=1}^{\infty} (\ln\ln(r+4))\,(r^3U(r))^{-1} < \infty$

(the model (1) with $1 < \gamma < 2$ readily satisfies these conditions) and proved that one can find a value of the inverse temperature β_1 such that if $\beta > \beta_1$ then there exist at least two extremal Gibbs states P^+ and P^- corresponding to the ground states $\phi(x) = +1$ and $\phi(x) = -1$. This very profound result is connected with the following fact. Let us consider the segment $[-n, n]$, the boundary conditions $\overline{\phi} = 1$ and the configuration $\phi^n_{-1}(x)$ such that $\phi^n_{-1}(x) = -1$, if $x \in [-n, n]$; $\phi^n_{-1}(x) = 1$, if $x \in \mathbf{Z}^1 - [-n, n]$. Then the difference between the energies of the configurations ϕ^n_{-1} and $\overline{\phi}$ has the order $n^{2-\gamma}$. In other words, in the one-dimensional case there is an analog of the surface tension and this fact leads to the existence of two extremal Gibbs states, as could be anticipated [4, 5]. In the borderline case $\gamma = 2$ the existence of phase transition was established by Frohlich and Spencer in [6]. Other sophisticated results in this borderline case concerning percolation density magnetization were obtained in [7, 8]. An alternative proof of the existence of phase transitions in ferromagnetic systems (1) for $1.5 \leqslant \gamma \leqslant 2$ based on geometric detailed descriptions of the spin configurations has recently been given in [9].

In this paper, we investigate the phase diagram of the model (1) under an additional external field. Consider a model with the following Hamiltonian:

$$H(\phi) = H_0(\phi) + \sum_{x \in \mathbf{Z}^1} h_x \phi(x). \tag{2}$$

Naturally, if the external field is sufficiently strong, it exterminates the pair interaction and the dependence on the boundary conditions disappears in the limit.

Theorem 1. *At any fixed value of the inverse temperature β there exists a constant h_0 such that for all realizations of the external field $\{h_x, x \in \mathbf{Z}^1\}$ satisfying $|h_x| > h_0, x \in \mathbf{Z}^1$ the model (2) has at most one limiting Gibbs state.*

Theorem 1 follows from the following theorem 2 which covers a more general case when the interaction potential is not specified. Consider a model on $\mathbf{Z}^1$ with the formal Hamiltonian

$$H_0(\phi) = \sum_{B \subset \mathbf{Z}^1} U(\phi(B)) \tag{3}$$

where the spin variables $\phi(x) \in \Phi$, Φ is a finite subset of the real line $\mathbf{R}$, $\phi(B)$ denotes the restriction of the configuration ϕ to the set B, the potential $U(\phi(B))$ is not necessarily translationally invariant. On the potential $U(\phi(B))$ we impose a natural condition, necessary for the existence of the thermodynamic limit:

$$\sum_{B \subset \mathbf{Z}^1 : x \in B} |U(\phi(B))| < C_0 \tag{4}$$

where the constant C_0 does not depend on x and the configuration ϕ. Now we consider random perturbations of the model (3), namely a model with the Hamiltonian

$$H(\phi) = H_0(\phi) + \sum_{x \in \mathbf{Z}^1} h_x \phi(x) \tag{5}$$

where $\{h_x, x \in \mathbf{Z}^1\}$ is a random external field.

Theorem 2 [10]**.** *For any model (3) and any fixed value of the inverse temperature β there exists a constant h_0 such that for all realizations of the random external field $\{h_x, x \in \mathbf{Z}^1\}$ satisfying $|h_x| > h_0, x \in \mathbf{Z}^1$ the model (5) has at most one limiting Gibbs state.*

The case of an external field with a small absolute value is open for all possibilities.

We consider perturbations of the model (1) with the periodic external field constituted by alternating (+) and (−) blocks:

$$H(\phi) = -\sum_{x,y\in\mathbf{Z}^1;x>y} U(x-y)\phi(x)\phi(y) + \sum_{x\in\mathbf{Z}^1} h_x^r\phi(x) \tag{6}$$

where h_x^r is a periodic function of period $2r$: $h_x^r = h_{x+2rk}$ for all integer values of k and for some fixed positive ϵ

$$h_x^r = \begin{cases} +\epsilon & \text{if} \quad x = 1,\ldots,r \\ -\epsilon & x = r+1,\ldots,2r. \end{cases}$$

The main result of the present paper is the following

Theorem 3. *Let ϵ be an arbitrary positive fixed number not exceeding some constant h_1. There exist natural numbers $R_1 = R_1(\epsilon)$ and $R_2 = R_2(\epsilon)$ such that at all sufficiently small temperatures the model (6) has at least two limiting Gibbs states for all $r \leqslant R_1$ and at most one limiting Gibbs state for all $r > R_2$.*

The value of h_1 will be given below.

2. Proof of theorem 3

Part 1. Now we prove that there exists a natural number R_2 such that for all $r > R_2$ at sufficiently small temperatures there is at most one limiting Gibbs state. Evidently for this part of theorem 3 the condition $\epsilon < h_1$ is not required.

For each natural number n let V_n be the interval $\left[\frac{1}{2} - r - rn, \frac{1}{2} + r + rn\right]$. We denote the set of all configurations $\phi(V_n)$ by Φ^n. Suppose that the boundary conditions ϕ^i, $i = 1, 2$, are fixed. The concatenation of the configurations $\phi(V_n)$ and $\phi^i(\mathbf{Z}^1 - V_n)$ we denote by χ : $\chi(x) = \phi(x)$, if $x \in V_n$ and $\chi(x) = \phi^i(x)$, if $x \in \mathbf{Z}^1 - V_n$. Define

$$H_n(\phi|\phi^i) = \sum_{B\subset\mathbf{Z}^1:B\cap V_n\neq\emptyset} U(\chi(B)).$$

Let $\phi_n^{\min,i} \in \Phi^n$ be a configuration with the minimal energy at fixed boundary conditions ϕ^i:

$$\min_{\phi\in\Phi^n} H_n(\phi|\phi^i) = H_n\left(\phi_n^{\min,i}\middle|\phi^i\right).$$

Define the following periodic configuration σ^r:

$$\sigma^r(x) = \begin{cases} +1 & \text{if } x = 1,\ldots,r \\ -1 & x = r+1,\ldots,2r \end{cases}$$

and $\sigma^r(x) = \sigma^r(x+2rk)$ for all integer values of k.

The following lemma describes the structure of the configuration $\phi_n^{\min,i}$. It turns out that at sufficiently large values of R in the configuration with the minimal energy, spins of all sites are directed along the external field (even for sites located very close to the boundary).

Lemma 1. *Let ϵ be any fixed positive number and the boundary conditions $\phi^i(\mathbf{Z}^1 - V_n)$ be fixed. A natural number R_2 exists such that if $r > R_2$ then the configuration $\phi_n^{\min,i}$ is independent of the boundary conditions ϕ^i: $\phi_n^{\min,i}(V_n) = \sigma^r(V_n)$.*

Let us consider an arbitrary configuration ϕ. We say that an interval $[k-1/2, k+1/2]$ is not regular, if $\phi([k-1/2, k+1/2]) \neq \sigma^r([k-1/2, k+1/2])$. Two non-regular cubes are called connected provided their intersection is not empty. The connected components of non-regular segments defined in such a way are called supports of contours and are denoted by supp K. A pair $K = (\text{supp } K, \phi(\text{supp } K))$ is called a contour. The set of all non-regular cubes we call a boundary of the configuration ϕ and denote by Γ. $|\Gamma|$ denotes the total length of all non-regular intervals.

It turns out that at large values of R the configuration with the minimal energy $\phi_n^{\min,i} = \sigma^r$ in lemma 1 is the Peierls stable ground state [11].

Lemma 2. *Let the boundary conditions $\phi^i(\mathbf{Z}^1 - V_n)$ be fixed and $\phi(V_n)$ be an arbitrary configuration. A natural number R_2 exists such that if $r > R_2$ then*

$$H_n(\phi|\phi^i) - H_n(\sigma^r|\phi^i) \geqslant \tau|\Gamma|$$

where a positive constant $\tau = \epsilon$ and Γ is a boundary of $\phi(V_n)$.

Obviously, lemma 1 is a immediate consequence of lemma 2. Let us prove the following auxiliary

Lemma 3. *Let the boundary conditions $\bar{\phi}(\mathbf{Z}^1 - [1, r])$ be fixed and $\phi([1, r])$ be an arbitrary configuration. A natural number R exists such that if $r \geqslant R_2$ then*

$$H_n(\phi|\bar{\phi}) - H_n(\sigma^r|\bar{\phi}) \geqslant \tau|\Gamma| \tag{7}$$

where a positive constant $\tau = \epsilon$ and Γ is a boundary of $\phi([1, r])$.

Proof. Let Γ be the boundary of the configuration $\phi([1, r])$. In other words, $\phi([1, r])$ is a perturbation of the constant configuration $\sigma^r([1, r]) = 1$ on all sites belonging to Γ. □

We choose a natural number N_1 such that $2\sum_{i=N_1+1}^{\infty} U(i) = 2\sum_{i=N_1+1}^{\infty} i^{-\gamma} < 1$ and define a real number M by

$$M = \max\left(N_1, \left(\frac{8}{(2-\gamma)\epsilon}\right)^{\frac{1}{\gamma-1}}\right).$$

The proof of the inequality (7) we divide into two cases.

Case 1. Large perturbations: $|\Gamma| > M$. We readily have

$$\begin{aligned} H_n(\phi|\bar{\phi}) - H_n(\sigma^r|\bar{\phi}) &\geqslant 2\epsilon|\Gamma| - 2 \sum_{x,y\in\mathbf{Z}^1; x\in\Gamma, y\in\mathbf{Z}^1-[1,r]} U(|x-y|) \\ &\geqslant \epsilon|\Gamma| + \epsilon|\Gamma| - 2\left(2\sum_{1}^{|\Gamma|} iU(i) + 2\sum_{|\Gamma|+1}^{\infty} U(i)\right). \end{aligned}$$

The last inequality is due to the fact that the term $U(i)$ with $i \leqslant |\Gamma|$ in $\sum_{x,y\in\mathbf{Z}^1; x\in\Gamma, y\in\mathbf{Z}^1-[1,r]} U(|x-y|)$ can appear at most $2i$ times. We estimate the first sum by integral estimation and note that since $|\Gamma| > N_1$ the second sum is less than 2 :

$$\begin{aligned} H_n(\phi|\bar{\phi}) - H_n(\sigma^r|\bar{\phi}) &\geqslant \epsilon|\Gamma| + \epsilon|\Gamma| - 2\left(2\left(1 + \int_1^{|\Gamma|} t^{1-\gamma}\,\mathrm{d}t\right) + 2\right) \\ &= \begin{cases} \epsilon|\Gamma| + \epsilon|\Gamma| - \left(8 + 4\dfrac{|\Gamma|^{2-\gamma} - 1}{2-\gamma}\right) & \text{if} \quad \gamma < 2 \\ \epsilon|\Gamma| + \epsilon|\Gamma| - (8 + 4\ln|\Gamma|) & \text{if} \quad \gamma = 2 \end{cases} \end{aligned}$$

Now note that since $|\Gamma| > M$ in both cases

$$H_n(\phi|\bar{\phi}) - H_n(\sigma^r|\bar{\phi}) \geqslant \epsilon|\Gamma|.$$

Thus, the inequality (7) is held with $\tau = \epsilon$.

Case 2. Small perturbations: $|\Gamma| \leqslant M$. Given ϵ choose a natural number $N = N_\epsilon$ such that $2\sum_{i=N_\epsilon+1}^{\infty} U(i) = 2\sum_{i=N_\epsilon+1}^{\infty} i^{-\gamma} < \epsilon$. Then

$$H_n(\phi|\bar{\phi}) - H_n(\sigma^r|\bar{\phi}) = \sum_{x\in\Gamma}\left(2\epsilon + \sum_{y\in[1,r]}(\sigma^r(x)\sigma^r(y) - \phi(x)\phi(y))|x-y|^{-\gamma}\right.$$
$$\left. + \sum_{y\in\mathbf{Z}^1-[1,r]}(\sigma^r(x)\bar{\phi}(y) - \phi(x)\bar{\phi}(y))|x-y|^{-\gamma}\right) = \sum_{x\in\Gamma}\left(2\epsilon + \sum\nolimits^1 + \sum\nolimits^2\right). \tag{8}$$

Obviously, all terms of $\sum^1$ are non-negative. Now to each negative term of $\sum^2$ with $|x-y| \leqslant N_\epsilon$ we assign a positive term of $\sum^1$ with the same absolute value such that different terms of $\sum^1$ will be assigned to different terms of $\sum^2$. Suppose that $(\sigma^r(x)\bar{\phi}(y) - \phi(x)\bar{\phi}(y))|x-y|^{-\gamma}$ is a negative term of $\sum^2$. Then the only possibility is: $\bar{\phi}(y) = -1$ and $\phi(x) = 1$. Let us define a sequence of lattice points v_m, $m \geqslant 1$, as follows: $v_1 = y$, $v_2 = x$, $v_m = T(v_{m-1}, v_{m-2})$ for $m > 2$, where $T(x, y) = 2x - y$ denotes the point which is symmetric to the point y with respect to the point x. Let k be a minimal index with positive value of $\phi(v_k)$: $k = \min_{\phi(v_i)=1} i$. Now to the term $(\sigma^r(x)\bar{\phi}(y) - \phi(x)\bar{\phi}(y))|x-y|^{-\gamma}$ we assign a term $(\sigma^r(v_{m-1})\sigma^r(v_m) - \phi(v_{m-1})\phi(v_m))|v_{m-1} - v_m|^{-\gamma}$ provided such k exists and v_k belongs to the interval $[1, r]$. We guarantee the condition $v_k \in [1, r]$ by choosing sufficiently large R: indeed, since $|x-y| \leqslant N_\epsilon$ the number of sites $y \in [1, r]$ with negative $\phi(y)$ is $|\Gamma|$ and is bounded by M. If $r \geqslant R_2 = N_\epsilon(M+1)$ then v_k is well defined and by construction the above-defined correspondence is one-to-one. As far as the remaining negative terms of $\sum^2$ with $|x-y| > N_\epsilon$, by definition of N_ϵ the absolute value of their sum is bounded by ϵ. Thus, $2\epsilon + \sum^1 + \sum^2 \geqslant \epsilon$ and

$$H_n(\phi|\bar{\phi}) - H_n(\sigma^r|\bar{\phi}) \geqslant \sum_{x\in\Gamma}\epsilon = \epsilon|\Gamma|.$$

Lemma 3 is proved.

Proof of lemma 2. Let us partition the segment $V_n = \left[\frac{1}{2} - r - rn, -\frac{1}{2} + r + rn\right]$ into $2n+1$ segments of length r : $V_n = \cup_{i=-n-1}^{n} I_i$, where $I_i = \left[\frac{1}{2} + ir, \frac{1}{2} + r + ir\right]$. We have

$$H_n(\phi|\phi^i) - H_n(\sigma^r|\phi^i) = \sum_{i=-n-1}^{n} E_i + \sum_{i\neq j; i,j=-n-1}^{n} E_{i,j} + \sum_{i=-n-1}^{n} E_{i,n}$$

where

$$E_i = \sum_{x,y\in\mathbf{Z}^1; x,y\in I_i; x>y} U(x-y)(\sigma^r(x)\sigma^r(y) - \phi(x)\phi(y)) + \sum_{x\in\mathbf{Z}^1; x\in I_i} h_x(\phi(x) - \sigma^r(x)),$$

$$E_{i,j} = \sum_{x,y\in\mathbf{Z}^1; x\in I_i; y\in I_j} U(|x-y|)(\sigma^r(x)\sigma^r(y) - \phi(x)\phi(y))$$

and

$$E_{i,n} = \sum_{x,y\in\mathbf{Z}^1; x\in I_i; y\in\mathbf{Z}^1-V_n} U(|x-y|)(\sigma^r(x)\sigma^r(y) - \phi(x)\phi(y)).$$

Now we define $A_i = E_i + E_{i,n} + \sum_{j:j>i} E_{i,j}$. In other words, we distribute terms $E_{i,j}$: we add $E_{i,j}$ either to E_i or to E_j. Actually this distribution can be carried out in any other way. Finally, we have

$$H_n(\phi|\phi^i) - H_n(\sigma^r|\phi^i) = \sum_{i=-n-1}^{n} A_i.$$

In order to prove the lemma 2 we prove the following inequality,

$$A_i \geqslant \tau|\Gamma_i|, \tag{9}$$

where $|\Gamma_i|$ is the length of the intersection of the support of the boundary Γ with the interval I_i. Without loss of generality we assume that $\sigma^r(I_i) = 1$. Now the inequality (9) is a consequence of lemma 3: as in the proof of lemma 3 we can expand A_i (as in (8)) :

$$A_i = \sum_{x\in\Gamma}\left(2\epsilon + \sum{}^{1} + \sum{}^{2}\right) \tag{10}$$

and again as in the proof of lemma 3 to each negative term of $\sum^2$ we can assign a positive term of $\sum_1$ (due to the definition of A_i the number of negative terms in $\sum^2$ of (10) is not greater than the number of negative terms in $\sum^1$ of (10)). Thus, lemma 2 is established with $R_2 = N_\epsilon(M + 1)$. □

Now we prove the uniqueness of the limiting Gibbs states in model (6). In our case the well-known uniqueness theorem [1–3] is not applicable: since the interaction has a very long range ($\gamma \leqslant 2$) the total interaction energy of the spins on two complementary half-lines is not finite. On the other hand, the fact that a one-dimensional model with translationally-invariant long-range interaction has a unique ground state cannot guarantee the absence of phase transition [12].

In order to prove the uniqueness of Gibbs states we use the method employing closed relationship between phase transitions and percolation in models with unique ground state [13]. The method uses the idea of 'coupling' of two independent partition functions and is based on the method used in [14]. Similar 'coupling' arguments are also at the center of the disagreement percolation approach to the Gibbs states uniqueness problem [15, 16]. The application of this theory to one-dimensional models at low temperatures produces the following uniqueness criterion [13].

We say that the ground state ϕ^{gr} of the model satisfies the Peierls stability condition, if there exists a constant t such that for any finite set $A \subset \mathbf{Z}^1$ $H(\phi') - H(\phi^{gr}) \geqslant t|A|$, where $|A|$ denotes the number of sites of A and ϕ' is a perturbation of ϕ^{gr} on the set A.

Condition 1. The only ground state ϕ^{gr} of the model satisfies the Peierls stability condition.

Condition 2. There exists a constant $\alpha < 1$ such that for any number L and any interval $I = [a, b]$ with the length n and for any configuration $\phi(I)$

$$\sum_{B\subset\mathbf{Z}^1;\, B\cap I\neq\emptyset,\, B\cap(Z^1-[a-L,b+L])\neq\emptyset} |U(B)| \leqslant \text{const}\, n^\alpha\, L^{\alpha-1}.$$

This condition is very natural and obviously is held for a pair potential $U(x - y) = (x - y)^{-\gamma}$ $(1 < \gamma \leqslant 2)$ of the model (6).

Theorem 4 [13]**.** *Suppose that a one-dimensional model with a finite spin space and with the translationally-invariant Hamiltonian*

$$H(\phi) = \sum_{B\subset\mathbf{Z}^1} U(\phi(B))$$

where $\sum_{B\subset \mathbf{Z}^1; x\in B} |U(B)| < const$ *satisfies the conditions 1 and 2. Then there exists a value of the inverse temperature* β_{cr} *such that if* $\beta > \beta_{cr}$ *then the model has at most one limiting Gibbs state.*

We can treat the model (6) as a translationally invariant model: it is well known that if we partition the lattice into disjoint intervals $Q(z)$ of length $2r$ centered at $z \in 2r\mathbf{Z}^1$ and replace the spin space $\{1, -1\}$ by $\{1, -1\}^Q$ including 2^{2r} elements, then the model from translationally periodic with period $2r$ transfers to the translationally invariant model. Therefore, we can apply theorem 3 in our case. Lemmas 1 and 2 provide that the model (6) satisfies the condition 1 and the first part of theorem 2 immediately follows from theorem 4.

Part 2. In order to prove the existence of a natural number R_1 such that the model (6) for all $r \leqslant R_1$ and at all sufficiently small temperatures has at least two limiting Gibbs states, we prove that if ϵ is a sufficiently small positive number and $r = 1$ then the model (6) has at least two limiting Gibbs states. In this case we have

$$\begin{aligned} H(\phi) &= \sum_{x,y\in\mathbf{Z}^1; x>y} U(x-y)\phi(x)\phi(y) + \sum_{x\in\mathbf{Z}^1} h_x^1\phi(x) \\ &= \sum_{x,y\in\mathbf{Z}^1; x=y+1; x \text{ is odd}} \left(U(1)\phi(x)\phi(y) + \frac{1}{2}\epsilon(\phi(x)-\phi(y))\right) + \sum_{x,y\in\mathbf{Z}^1; x=y+1; x \text{ is even}} \\ &\quad\times \left(U(1)\phi(x)\phi(y) - \frac{1}{2}\epsilon(\phi(x)-\phi(y))\right) + \sum_{x,y\in\mathbf{Z}^1; x>y+1} U(x-y)\phi(x)\phi(y) \\ &= \sum_{x,y\in\mathbf{Z}^1; x>y} \bar{U}(x-y)\phi(x)\phi(y) \end{aligned}$$

where $\bar{U}(k) = U(k)$ for $k \geqslant 2$ and $\bar{U}(1) = U(1) \pm \frac{\epsilon(\phi(x)-\phi(y))}{2\phi(x)\phi(y)}$. In other words, we incorporate the half of the external field h_x into the interaction between neighboring spins $\phi(x-1)$ and $\phi(x)$ and the other half of the external field h_x into the interaction between neighboring spins $\phi(x)$ and $\phi(x+1)$. The new potential $\bar{U}(1)$ becomes $U(1), U(1)+\epsilon$ or $U(1)-\epsilon$ depending on the parity of x and the values of spins at x and y. Let us choose h_1 such that $h_1 \leqslant U(1) - U(2)$. Then the model remains ferromagnetical: $\bar{U} > 0$. Actually, the potential $\bar{U}(1)$ is a constant potential $U(1)-\epsilon$ plus some nonnegative correction taking values 0, ϵ or 2ϵ. Also due to the condition $h_1 \leqslant U(1) - U(2)$ the new potential is monotonically decreasing: $\bar{U}(k) < \bar{U}(2) < \bar{U}(1)$ in all cases and for all values of $k \geqslant 3$. Thus, for all $\epsilon < h_1$ the obtained Hamiltonian is ferromagnetical and has two Peierls stable ground states: constant configurations $\phi = 1$ and $\phi = -1$. The statement of the theorem 2 now directly follows from [4, 5] for $1 < \gamma < 2$ and from [6] or [9] for $\gamma = 2$. Theorem 3 is proved.

3. Concluding remarks

(1) One-dimensional models with combined ferromagnetic, antiferromagnetic short or long-range interactions and external fields exhibit many expected and unexpected interesting results [17].

(2) We expect that the effect of the external field is 'monotonic' with respect to the block size; in other words, the values of R_1 and R_2 coincide. But the methods of this paper do not allow us to prove this statement.

(3) It follows from the proof of theorem 2 that if the block size exceeds R_2, then instead of blocks with alternating signs we can consider any order of + and − blocks of sizes

exceeding R_2 and the second part of theorem 2 will be still held: at sufficiently low temperatures the limiting Gibbs state will be unique.

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